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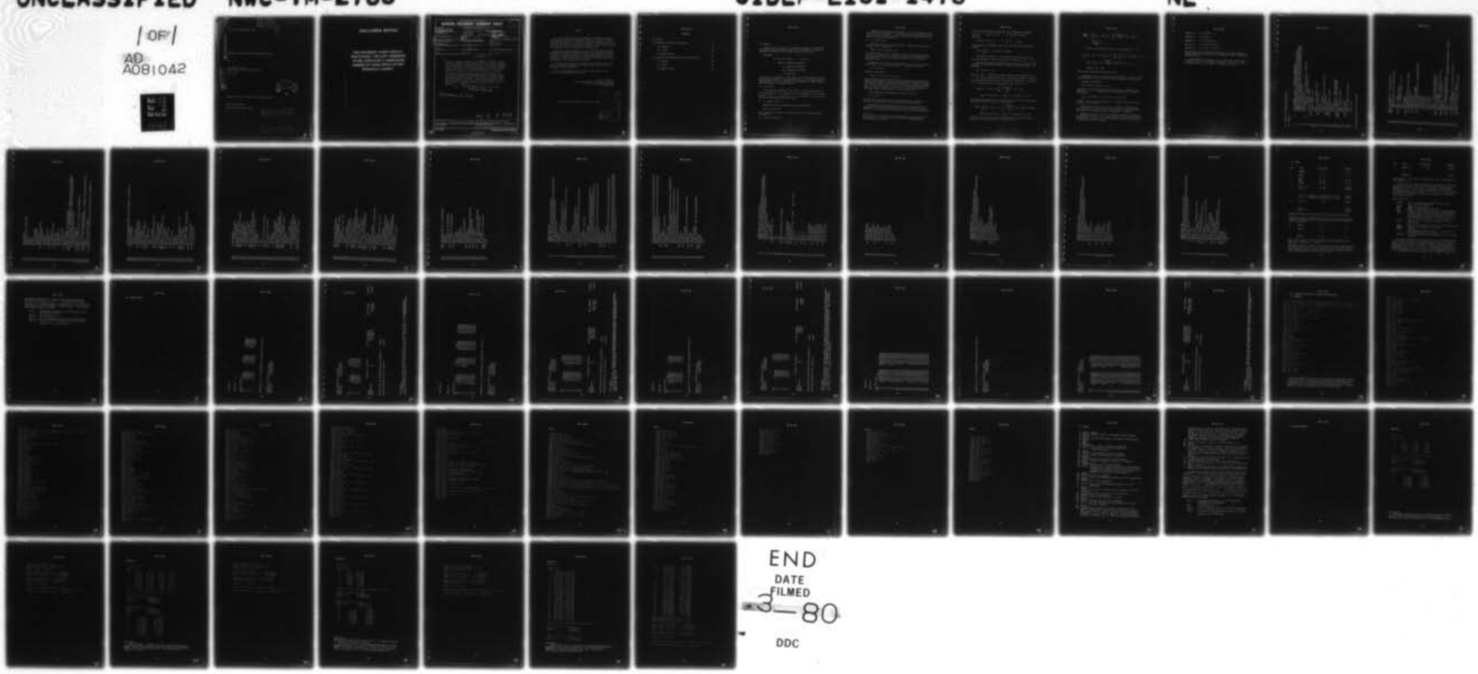

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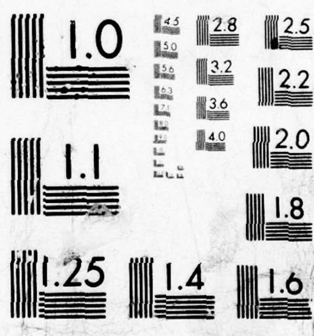

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NONLINEAR REGRESSION THEORY AND COMPUTER PROGRAMS

by

Dr. Barbara J. Bell
Weapon Systems Cost Analysis Division

April 1976

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Naval Weapons Center

China Lake, California 93555

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The Naval Weapons Center Weapon Systems Cost Analysis Division develops Cost Estimating Relationships (CERs) to estimate the relationships between the costs of current and past weapon systems to system physical, performance, and technical parameters. These CERs can be used to predict the costs of future weapons systems. CERs are often regression equations that are nonlinear in the parameters.

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FOREWORD

The Naval Weapons Center Weapon Systems Cost Analysis Division develops Cost Estimating Relationships (CERs) to estimate the relationships between the costs of current and past weapon systems to system physical, performance, and technical parameters. These CERs can be used to predict the costs of future weapons systems. CERs are often regression equations that are nonlinear in the parameters.

This report presents programs in the FORTRAN V and BASIC computer languages employing a statistical technique to estimate the parameters for nonlinear regression equations. These computer programs could be modified or expanded in the future to utilize an improved statistical technique or to include additional statistical calculations and output that would provide information to the user.

The study reported herein was conducted from October 1974 to June 1975. The computer programs were revised in March 1976 to present additional output information.

This is an informal report of the Naval Weapons Center and is not to be used as authority for action.

Floyd W. Erickson
Head, Weapon Systems Cost Analysis Division
Office of Finance and Management
30 March 1976

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CONTENTS

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1.0 THEORY

The purpose of this study was to develop programs, in the FORTRAN V and BASIC computer languages, employing a statistical technique to estimate the parameters for nonlinear regression equations.

The model

$$Y = \Theta_0 + \Theta_1 X_1 + \Theta_2 X_2 + \cdot\cdot\cdot + \Theta_p X_p + \varepsilon$$

where Y = Dependent variable

Θ_j = Regression parameter

X_h = Independent variable

ε = Error term

is linear in the parameters. This model can be fitted using the linear least squares technique. Any model which is not of the above form is a nonlinear model, that is, nonlinear in the parameters. The following are some examples of nonlinear models.

1) $Y = e^{\Theta} X + \varepsilon$

This model has an additive error term. It can be made linear by the transformation of the parameter. If we transform $\beta = e^{\Theta}$, the model becomes $Y = \beta X + \varepsilon$. The model $Y = e^{\Theta} + \varepsilon$ is intrinsically linear since it can be transformed into a linear form.

2) $Y = \exp(\Theta_1 + \Theta_2 X + \varepsilon)$

This model can be transformed into the linear model

$$\ln Y = \Theta_1 + \Theta_2 X + \varepsilon$$

It is intrinsically linear. The error term in the transformed model is additive.

3) $Y = \Theta_1 X_1^{\Theta_2} X_2^{\Theta_3} + \varepsilon$

This model is intrinsically nonlinear. It is not possible to convert it into a linear model. A ln transformation cannot be made because the error term is additive. Linear least squares techniques cannot be used to estimate the parameters.

4) $Y = \Theta_1 X_1^{\Theta_2} X_2^{\Theta_3} \varepsilon$

The error term here is multiplicative. This model can be transformed into the linear model

$$\ln Y = \ln \Theta_1 + \Theta_2 \ln X_2 + \ln \varepsilon$$

It is intrinsically linear. However, fitting this transformed model by linear least squares techniques will result in estimated parameters that minimize the sum of the squared deviations of ln ε from the regression line.

Examples 3 and 4 point out the common error of ignoring the form of the error term in a nonlinear model. For example, the model

$$Y = \Theta_1 X_1^{\Theta_2} X_2^{\Theta_3}$$

would be transformed to

$$\ln Y = \ln \Theta_1 + \Theta_2 \ln X_1 + \Theta_3 \ln X_2$$

This transformed model would be fitted by standard least squares techniques, and it would be assumed that the sum of the squared deviations of ε from the regression line would be minimized. As shown in Example 3, if the error term were additive, the ln transformation could not be made. As shown in Example 4, if the error term were multiplicative, the model would be fitted using ln ε as the error term.

Cost estimating relationships are often of the form

$$Y = \Theta_1 X_1^{\Theta_2} + \varepsilon$$

Thus it would be of value to have a valid statistical technique to estimate the parameters of this intrinsically nonlinear model.

One method of fitting the parameters in a nonlinear regression equation is the iterative modified Gauss-Newton least squares method[1]. Consider the following nonlinear problem: Given n sets of observations

[1]H. O. Hartley. "The Modified Gauss-Newton Method for the Fitting of Nonlinear Regression Functions by Least Squares", Technometrics, Vol. 3 (1961), pp. 269-80.

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each with one dependent variable Y_h, and 1 independent variables, $X_{h1}, X_{h2}, \cdots X_{hl}$, $(h = 1,2, \cdots n)$, and the nonlinear function

$$f\,(X,\Theta) = f\,(X_1, X_2, \cdots X_l\,;\,\Theta_1, \Theta_2, \cdots \Theta_m)$$

it is required to determine a set of Θ_k for which the error sums of squares

$$Q\,(\Theta) = \sum_{h=1}^{n} (Y_h - f\,(X_h, \Theta))^2 = \text{Minimum}$$

The symbol X_h stands for the h-vector with elements $X_{h1}, X_{h2}, \cdots X_{hl}$ and the symbol Θ for the m-vector with elements $\Theta_1, \Theta_2, \cdots \Theta_m$.

The Modified Gauss-Newton method uses the results of linear least squares in a succession of stages. Suppose the postulated model is of the form

$$Y_h = f\,(X_h, \Theta) + \varepsilon_h$$

Let ${}_0\Theta_1, {}_0\Theta_2, \cdots {}_0\Theta_m$ be the initial values of the parameters $\Theta_1, \Theta_2, \cdots \Theta_m$. These initial values may be intelligent guesses or preliminary estimates based on whatever information is available. If we carry out a first-order Taylor's expansion of $f\,(X_h, \Theta)$ at $\Theta = {}_0\Theta$,

$$f\,(X_h, \Theta) = f\,(X_h, {}_0\Theta) + \sum_{j=1}^{m} \frac{\partial f\,(X_h, \Theta)}{\partial\Theta_j} (\Theta_j - {}_0\Theta_j)$$

where the derivative of $f(X_h, \Theta)$ is evaluated at $\Theta = {}_0\Theta$.

This linear approximation to the nonlinear function can be substituted into the error sums of squares, $Q(\Theta)$, equation

$$Q(\Theta) = \sum_{h=1}^{n} \{Y_h - f(X_h, {}_0\Theta) - \sum_{j=1}^{m} \frac{\partial\, f(X_h, \Theta)}{\partial\,\Theta_j} (\Theta_j - {}_0\Theta_j)\} = \text{Minimum}$$

where the derivative of $f(X_h, \Theta)$ is evaluated at $\Theta = {}_0\Theta$.

The value of the vector Θ that minimizes $Q(\Theta)$ is obtained from the solution of the following set of linear equations.

$$\frac{\partial Q(\Theta)}{\partial \Theta_i} = -2 \sum_{h=1}^{n} \{Y_h - f(X_h, {}_0\Theta) - \sum_{j=1}^{m} \frac{\partial f(X_h, \Theta)}{\partial \Theta_j} (\Theta_j - {}_0\Theta_j)\} \cdot$$

$$\frac{\partial f(X_h, \Theta)}{\partial \Theta_i} = 0$$

where the derivative of $f(X_h, \Theta)$ is evaluated at $\Theta = {}_0\Theta$.

$$\sum_{h=1}^{n} \{Y_h - f(X_h, {}_0\Theta)\} f_i(X_h, {}_0\Theta) = \sum_{j=1}^{m} \{\sum_{h=1}^{n} f_i(X_h, {}_0\Theta) f_j(X_h, {}_0\Theta)\} D_j$$

where $f_i(X_h, {}_0\Theta) = \frac{\partial f(X_h, \Theta)}{\partial \Theta_i}$ evaluated at $\Theta = {}_0\Theta$

and $D_j = (\Theta_j - {}_0\Theta_j)$

These normal equations can be solved for D_j.

We replace ${}_0\Theta$ by ${}_0\Theta + \frac{1}{2}D$ and evaluate $Q({}_0\Theta + \frac{1}{2}D)$. If $Q({}_0\Theta + \frac{1}{2}D)$ is less than $Q({}_0\Theta)$, ${}_0\Theta + \frac{1}{2}D$ is taken as the new estimate of the vector Θ.

Consider the function

$Q(v) = Q({}_0\Theta + vD)$ for $0 \le v \le 1$

and denote by v' the value for v for which $Q(v)$ is a minimum on the interval $0 \le v \le 1$. We approximate v' by evaluating $v = 0$, $\frac{1}{2}$ and 1 and solving

$$v_{min} = \tfrac{1}{2} + \tfrac{1}{4}\{Q(0) - Q(1)\} / \{Q(1) - 2Q(\tfrac{1}{2}) + Q(0)\}$$

If $Q({}_0\Theta + \frac{1}{2}D)$ is not less than $Q({}_0\Theta)$, ${}_0\Theta + v_{min}D$ is taken as the new estimate of the vector Θ.

The iteration is continued until the maximum fractional change in the parameter estimates is less than or equal to a specified tolerance level, or the desired number of iterations is completed.

The above method is utilized in a FORTRAN V language program for the UNIVAC 1110 computer and a BASIC language program for the Hewlett-Packard 9830A calculator. The following nonlinear equations have been incorporated into these programs.

Equation 1) $Y = \Theta_1 e^{\Theta_2 X} + \varepsilon$

Equation 2) $Y = \Theta_1 X^{\Theta_2} + \varepsilon$

Equation 3) $Y = \Theta_1 X_1^{\Theta_2} X_2^{\Theta_3} + \varepsilon$

Equation 4) $Y = \Theta_1 X_1^{\Theta_2} X_2^{\Theta_3} X_3^{\Theta_4} + \varepsilon$

Equation 5) $Y = \Theta_1 X_1^{\Theta_2} + \Theta_3 X_2^{\Theta_4} + \varepsilon$

The parameters for two additional equations, each with a maximum of seven independent variables and seven parameters, can be estimated as needed by the user.

The programs are designed to stop iteration when the maximum fractional change in the parameter estimates is less than or equal to a tolerance input, or the number of iterations equals 15.

2.0 UNIVAC 1110 COMPUTER PRESENTATION

2.1 PROGRAM *

```
      C     DR. BARBARA J. BELL - NONLINEAR REGRESSION
            DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7),B(7),Q0,Q5,Q1,
           1QTMIN,AS,BS,D(7),DA(7),DD(7),TSS,RSS,RSQ,CT,RMS,EMS,F,STDER
            COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
           1,NNX(7),NOBS,Y(30),IFNAL
            DIMENSION PX(7)
      C     READ NPRINT=0 - PRINT-OUT OF FINAL ITERATION
      C          NPRINT=1 - FULL PRINT-OUT
            READ (5,1002) NPRINT
       1002 FORMAT (10I5)
            READ (5,1002) NDATA
            ND=0
          5 READ (5,900) NOBS,NX,TOLER
        900 FORMAT (2I5,F5.0)
          1 ND=ND+1
            DO 10 I=1,NOBS
            READ (5,920) (X(I,J),J=1,NX),Y(I)
        920 FORMAT (8F10.0)
         10 CONTINUE
       3999 READ (5,1002) NEQUAT
            NITER=0
            IFNAL=0
            IF (NEQUAT-0)  1000,4100,1000
       1000 IF (NEQUAT-3) 1001,1003,1004
       1001 NTHETA=2
            READ (5,1002) NNX(1)
            NJ=1
            GO TO 1010
       1003 NTHETA=3
            READ (5,1002) NNX(1),NNX(2)
            NJ=2
            GO TO 1010
```

* Storage requirement of 10126 words.

```
 1004 IF (NEQUAT-5)  1005,1005,1006
 1005 NTHETA=4
      READ (5,1002) NNX(1),NNX(2),NNX(3)
      NJ=3
      GO TO 1010
 1006 READ (5,1002) NTHETA
      READ  (5,1002) (NNX(J), J=1,7)
      NJ=7
      GO TO 1010
 1010 NTA=NTHETA+1
      READ(5,910) (THETA(K),K=1,NTHETA)
  910 FORMAT (8F10.0)
      WRITE(6,939)
  939 FORMAT('1')
      WRITE (6,901) NEQUAT
  901 FORMAT ('1EQUATION',I5,//,'0OBSERVATIONS')
      PX(1)='X(1)'
      PX(2)='X(2)'
      PX(3)='X(3)'
      PX(4)='X(4)'
      PX(5)='X(5)'
      PX(6)='X(6)'
      PX(7)='X(7)'
      PY='Y'
      WRITE (6,2000) (PX(J),J=1,NX), PY
 2000 FORMAT (' NO.',9X,A4,7(11X,A4))
      DO 11 I=1,NOBS
      WRITE (6,2001) I,(X(I,J),J=1,NX),Y(I)
 2001 FORMAT (' ',I2,1X,8F15.5)
   11 CONTINUE
      WRITE (6,2002) (NNX(J),J=1,NJ)
 2002 FORMAT ('0'/'0NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED=',
     17I2)
      WRITE (6,938)
  938 FORMAT ('0'/'0INITIAL ESTIMATES OF THETA')
      DO 12 K=1,NTHETA
   12 WRITE (6,603) K, THETA(K)
  603 FORMAT (' THETA(',I1,')=',F20.10)
```

```
      CALL FUNCT
      IF (NPRINT.EQ.1) CALL RITE
      QD=0.0
      TSS=0.0
      CT=0.0
      SY=0.0
      DO 20 I=1,NOBS
      TSS=TSS+Y(I)**2
      SY=SY+Y(I)
   20 QD=QD+(Y(I)-FZ(I))**2
      CT=(SY**2)/NOBS
      TSS=TSS-CT
      M2=NOBS-1
      M3=NOBS-NTHETA-1
   25 CONTINUE
      NITER=NITER+1
      IF (NPRINT) 27,27,24
   24 RSS=TSS-QD
      RSQ=RSS/TSS
      RMS=RSS/NTHETA
      EMS=QD/M3
      IF (QD) 26,26,23
   23 F=RMS/EMS
      STDER=EMS**.5
   26 WRITE (6,2100) M2,TSS,NTHETA,RSS
 2100 FORMAT ('0',//,' ',15X,'DEGREES OF FREEDOM',11X,'SUM OF SQUARES'
     1,14X,'MEAN SQUARE',8X,'F-RATIO',/,' TOTAL',17X,I5,11X,F20.5,/,
     2' REGRESSION',12X,I5,11X,F20.5)
      IF (QD) 28,29,29
   28 WRITE (6,2101)
 2101 FORMAT ('0ERROR SUMS OF SQUARES ARE NEGATIVE')
      GO TO 27
   29 WRITE (6,2102) RMS,F,M3,QD,EMS
 2102 FORMAT  ('+', 63X,F20.5,5X,F10.3,/,' ERROR',17X,I5,11X,F20.5,5X,
     1F20.5)
      WRITE (6,2103) RSQ,STDER
 2103 FORMAT ('0',//,' COEFFICIENT OF DETERMINATION (R-SQUARE) =',
```

```
     1F10.5,/,'0STANDARD ERROR OF THE ESTIMATE =',4X,F15.5)
  27 DO 40 J=1,NTHETA
     DO 40 K=1,NTHETA
     AS=0.0
     DO 30 I=1,NOBS
  30 AS=AS+DFZ(I,K)*DFZ(I,J)
     IF (NPRINT) 40,40,31
  31 WRITE (6,939)
     WRITE (6,591) AS
 591 FORMAT (' AS=',F20.5)
  40 A(K,J)=AS
     DO 60 K=1,NTHETA
     BS=0.0
     DO 50 I=1,NOBS
  50 BS=BS+(Y(I)-FZ(I))*DFZ(I,K)
  60 B(K)=BS
     IF (NPRINT) 62,62,61
  61 DO 585 K=1,NTHETA
 585 WRITE (6,592) K,B(K)
 592 FORMAT (' B(',I2,')=',F20.5)
  62 DO 70 K=1,NTHETA
  70 A(K,NTA)=B(K)
     DO 71 K=1,NTHETA
  71 KR(K)=K
     DO 72 J=1,NTA
  72 KC(J)=J
     ISTAGE=0
     N=NTHETA
 401 CALL POSIT (ISTAGE,N, NTA)
 406 ISTAGE=ISTAGE+1
 593 FORMAT (' ',I5)
     CALL JORDAN (NTA)
     IF (NTHETA-ISTAGE) 450,450,400
 400 N=N-1
     IF (N-1) 406,406,401
 450 CALL ORIENT (NTA)
     DO 80 K=1,NTHETA
```

```
   80 D(K)=A(K,1)
      IF (NPRINT) 82,82,81
   81 DO 586 K=1,NTHETA
  586 WRITE (6,600) K,D(K)
  600 FORMAT (' D(',I2,')=',F20.5)
   82 DO 90 K=1,NTHETA
      DA(K)=D(K)/THETA(K)
   90 DD(K)=ABS(DA(K))
      IF (NPRINT) 89,89,88
   88 DO 587 K=1,NTHETA
  587 WRITE (6,601) K,DD(K)
  601 FORMAT (' DD(',I2,')=',F20.5)
   89 IF (NTHETA-1) 91,91,92
   91 AMXD=DD(1)
      GO TO 98
   92 K=2
      I=1
   93 IF (DD(I)-DD(K)) 94,95,95
   94 I=K
   95 IF (NTHETA-K) 96,96,97
   97 K=K+1
      GO TO 93
   96 AMXD=DD(I)
      IF (NPRINT) 98,98,101
  101 WRITE (6,602) AMXD
  602 FORMAT (' AMXD=',F20.10)
   98 IF (AMXD-TOLER) 4000,4000,110
  110 DO 120 K=1,NTHETA
  120 THETA(K)=THETA(K)+.5*D(K)
      CALL FUNCT
      Q5=0.0
      DO 130  I=1,NOBS
  130 Q5=Q5+(Y(I)-FZ(I))**2
      IF (NPRINT.EQ.1) CALL RITE
      IF (NPRINT) 132,132,131
  131 WRITE (6,604) Q5
  604 FORMAT (' Q5=', F20.5)
```


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```
132 IF (Q5-Q0) 151,150,150
150 DO 160 K=1,NTHETA
160 THETA(K)=THETA(K)+.5*D(K)
    CALL FUNCT
    IF (NPRINT.EQ.1) CALL RITE
    Q1=0.0
    DO 170 I=1,NOBS
170 Q1=Q1+(Y(I)-FZ(I))**2
    IF (NPRINT) 172,172,171
171 WRITE (6,605) Q1
605 FORMAT (' Q1=', F20.5)
172 TMIN=.5+.25*(Q0-Q1)/(Q0+Q1-2*Q5)
    IF (NPRINT) 174,174,173
173 WRITE (6,606) TMIN
606 FORMAT (' TMIN=',F15.5)
174 IF (TMIN) 4015,4000,4011
4011 IF (TMIN-1.0) 4012,4012,4015
4015 TMIN=1.0
    GO TO 4002
4012 CONTINUE
    DO 180 K=1,NTHETA
180 THETA(K)=THETA(K)-(1.0-TMIN)*D(K)
4007 CONTINUE
    CALL FUNCT
    IF (NPRINT.EQ.1) CALL RITE
    QTMIN=0.0
    DO 190 I=1,NOBS
190 QTMIN=QTMIN+(Y(I)-FZ(I))**2
    IF (NPRINT) 192,192,191
191 WRITE (6,607) QTMIN
607 FORMAT (' QTMIN=',F20.5)
192 IF (Q0-QTMIN) 4002,4002,4001
4002 CONTINUE
    DO 4008 K=1,NTHETA
```


15

```
 4008 THETA(K)=THETA(K)+(.5-TMIN)*D(K)
  151 CONTINUE
      QO=Q5
      IF (NITER-15) 25,4000,4000
 4001 QO=QTMIN
      IF (NITER-15) 25,4000,4000
 4000 IFNAL=1
      CALL RITE
      RSS=TSS-QO
      RSQ=RSS/TSS
      RMS=RSS/NTHETA
      EMS=QO/M3
      IF (QO) 4016,4016,4017
 4017 F=RMS/EMS
      STDER=EMS**.5
 4016 WRITE (6,2100) M2,TSS,NTHETA,RSS
      IF (QO) 3998,3998,4009
 3998 WRITE (6,2101)
      GO TO 3999
 4009 WRITE (6,2102) RMS,F,M3,QO,EMS
      WRITE (6,2103) RSQ,STDER
      GO TO 3999
 4100 IF (NDATA-ND) 4200,4200,5
 4200 STOP
      END
```

```
      SUBROUTINE FUNCT
      DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7)
      COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
     1,J1,J2,J3,J4,J5,J6,J7,NOBS,Y(30),IFNAL
      IF (NEQUAT-2) 1,2,3
    1 CONTINUE
      DO 501 I=1,NOBS
      FZ(I) =THETA(1)*EXP(THETA(2)*X(I,J1))
      DFZ(I,1)=EXP(THETA(2)*X(I,J1))
      DFZ(I,2)=THETA(1) *EXP(THETA(2)*X(I,J1))* X(I,J1)
  501 CONTINUE
      GO TO 1000
    2 CONTINUE
      DO 502 I=1,NOBS
      FZ(I)=THETA(1)*X(I,J1)**THETA(2)
      DFZ(I,1)=X(I,J1)**THETA(2)
      DFZ(I,2)=THETA(1)*X(I,J1)**THETA(2)*ALOG(X(I,J1))
  502 CONTINUE
      GO TO 1000
    3 IF (NEQUAT-4) 103,104,5
  103 CONTINUE
      DO 503 I=1,NOBS
      FZ(I)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)
      DFZ(I,1)=X(I,J1)**THETA(2)*X(I,J2)**THETA(3)
      DFZ(I,2)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)
     1*ALOG(X(I,J1))
      DFZ(I,3)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)
     1*ALOG(X(I,J2))
  503 CONTINUE
      GO TO 1000
  104 CONTINUE
      DO 504 I=1,NOBS
      FZ(I)=THETA(1!*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)*X(I,J3)
     1**THETA(4)
      DFZ(I,1)=X(I,J1)**THETA(2)*X(I,J2)**THETA(3)*X(I,J3)**THETA(4)
      DFZ(I,2)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)*X(I,J3)
     1**THETA(4)*ALOG(X(I,J1))
```


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```
      DFZ(I,3)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)*X(I,J3)
     1**THETA(4)*ALOG(X(I,J2))
      DFZ(I,4)=THETA(1)*X(I,J1)**THETA(2)*X(I,J2)**THETA(3)*X(I,J3)
     1**THETA(4)*ALOG(X(I,J3))
  504 CONTINUE
      GO TO 1000
    5 IF (NEQUAT-6) 105,106,107
  105 CONTINUE
      DO 505 I=1,NOBS
      FZ(I)=THETA(1)*X(I,J1)**THETA(2)+THETA(3)*X(I,J2)**THETA(4)
      DFZ(I,1)=X(I,J1)**THETA(2)
      DFZ(I,2)=THETA(1)*X(I,J1)**THETA(2)*ALOG(X(I,J1))
      DFZ(I,3)=X(I,J2)**THETA(4)
      DFZ(I,4)=THETA(3)*X(I,J2)**THETA(4)*ALOG(X(I,J2))
  505 CONTINUE
      GO TO 1000
  106 CONTINUE
      DO 506 I=1,NOBS
C     INSERT EQUATION 6 AND ITS DERIVATIVES HERE
  506 CONTINUE
      GO TO 1000
  107 CONTINUE
      DO 507 I=1,NOBS
C     INSERT EQUATION 7 AND ITS DERIVATIVES HERE
  507 CONTINUE
 1000 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE POSIT (ISTAGE,N,NTA)
      DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7),CA(7,8),BA(7,8)
      COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
     1,J1,J2,J3,J4,J5,J6,J7,NOBS,Y(30),IFNAL
      DIMENSION KKR(7),KKC(8)
      DO 55 I=1,NTHETA
      DO 55 J=1,NTA
   55 BA(I,J)=ABS(A(I,J))
      K=1
      L=2
      I=1
      J=1
   57 IF(BA(I,J)-BA(K,L)) 59,58,58
   59 I=K
      J=L
   58 IF (N-L) 90,61,62
   61 IF(N-K) 90,68,63
   90 WRITE(6,91) N
   91 FORMAT (' N=',I2,'  N LESS    THAN L OR K')
      GO TO 30
   63 K=K+1
      L=1
      GO TO 57
   62 L=L+1
      GO TO 57
   68 KK=I
      LL=J
      DO 69 I=1,NTHETA
      DO 69 J=1,NTA
   69 CA(I,J)=A(I,J)
      DO 65 J=1,NTA
      A(1,J)=CA(KK,J)
   65 A(KK,J)=CA(1,J)
      DO 70 I=1,NTHETA
      DO 70 J=1,NTA
   70 CA(I,J)=A(I,J)
      DO 66 I=1,NTHETA
```

```
      A(I,1)=CA(I,LL)
   66 A(I,LL)=CA(I,1)
      I1=1+ISTAGE
      I2=KK+ISTAGE
      DO 31 I=1,NTHETA
   31 KKR(I)=KR(I)
      KR(I1)=KKR(I2)
      KR(I2)=KKR(I1)
      K1=1+ISTAGE
      K2=LL+ISTAGE
      DO 32 J=1,NTA
   32 KKC(J)=KC(J)
      KC(K1)=KKC(K2)
      KC(K2)=KKC(K1)
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE JORDAN (NTA)
      DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7),W(8)
      COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
     1,J1,J2,J3,J4,J5,J6,J7,NOBS,Y(30),IFNAL
      DO 70 J=1,NTHETA
   70 W(J)=A(1,J+1)/A(1,1)
      W(NTA)=1.0/A(1,1)
      NNTA=NTHETA-1
      DO 72 I=1,NNTA
      DO 71 J=1,NTHETA
   71 A(I,J)=A(I+1,J+1)-A(I+1,1)*W(J)
   72 A(I,NTA)=0.0-A(I+1,1)*W(NTA)
      DO 73 J=1,NTA
   73 A(NTHETA,J)=W(J)
      RETURN
      END
```

```
      SUBROUTINE ORIENT (NTA)
      DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7),E(7,8)
      COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
     1,J1,J2,J3,J4,J5,J6,J7,NOBS,Y(30),IFNAL
      DO 81 I=1,NTHETA
      DO 81 J=1,NTA
   81 E(I,J)=A(I,J)
      DO 76 I=1,NTHETA
      M=KC(I)
      DO 76 J=1,NTA
   76 A(M,J)=E(I,J)
      DO 46 I=1,NTHETA
      DO 46 J=1,NTA
   46 E(I,J)=A(I,J)
      DO 77 J=1,NTHETA
      MM=KR(J)
      DO 77 I=1,NTHETA
   77 A(I,MM+1)=E(I,J+1)
      RETURN
      END
```

```
      SUBROUTINE RITE
      DOUBLE PRECISION A(7,8),FZ(30),DFZ(30,7)
      COMMON X(30,7),THETA(7),FZ,DFZ,A,KR(7),KC(8),NEQUAT,NTHETA
     1,J1,J2,J3,J4,J5,J6,J7,NOBS,Y(30),IFNAL
      IF (IFNAL) 6,6,7
    7 WRITE (6,9)
    9 FORMAT ('1')
      WRITE (6,8)
    8 FORMAT (' FINAL ESTIMATES OF THETA')
    6 DO 10 K=1,NTHETA
   10 WRITE (6,11) K,THETA(K)
   11 FORMAT (' THETA(',I1,')=',F20.10)
      WRITE (6,12)
   12 FORMAT ('0'/'0NO.',9X,'Y',14X,'Y-HAT')
      DO 15 I=1,NOBS
      WRITE (6,16) I,Y(I),FZ(I)
   16 FORMAT (' ',I2,1X,2F15.5)
      IF (NPRINT) 20,20,17
   17 DO 21 K=1,NTHETA
      WRITE (6,18) NTHETA,DFZ(I,K)
   18 FORMAT (' DERIVATIVE(',I1,')=',F15.5)
   21 CONTINUE
   20 CONTINUE
   15 CONTINUE
      RETURN
      END
```


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2.2 INPUTS

Card	Input	Card Column	Format*
1	NPRINT	1 - 5	I 5
2	NDATA	1 - 5	I 5
3	NOBS	1 - 5	I 5
	NX	6 - 10	I 5
	TOLER	11 - 15	F 5.0
4	X (1 , 1)	1 - 10	F 10.0
	X (1 , 2)	11 - 20	F 10.0
	.		.
	.		.
	.		.
	X (1 , J)	maximum of 7 independent variables	F 10.0
	Y (1)	immediately following last X (1,J)	F 10.0
5	X (2 , 1)	1 - 10	F 10.0
	.		.
	.		.
	.		.
	X (2 , J)		F 10.0
	Y (2)		F 10.0

Continue to place one observation per card, up to 30 observations.

Following the input of observations for each data set, three cards are used for the specifications of each equation for which the parameters are to be estimated.

Card	Input	Card Column	Format*
1	NEQUAT	1 - 5	I 5
2	NNX (1)	1 - 5	I 5
	.		.
	.		.
	NNX (7)	31 - 35	I 5

List in any order on Card 2 the independent variable number(s) to be used, up to a maximum of 7 variables.

*All inputs using the I format are integers that must be right-justified in their respective fields. All inputs using the F format are floating point numbers that must be right-justified in their respective fields unless a decimal point is included in the field.

Card	Input	Card Column	Format
3	THETA (1)	1 - 10	F 10.0
	.		.
	.		.
	.		.
	THETA (K)		F 10.0

List on Card 3, in the order corresponding to Card 2, the estimate of each parameter, Θ_k.

These three cards will be repeated for each equation. When these cards have been placed for all desired equations for the data set, place a card with 0 in card column 5. If NDATA is greater than 1, following the 0 card place in the following order: a card with NOBS, NX, and TOLER; the cards with X(I, 1), · · · X (I, J), Y (I); and three cards for each equation to be fitted for that data set. Repeat for the remaining data sets.

Input definitions:

- NPRINT - Set = 0 for print-out of final iteration only.
  - Set = 1 for full print-out.
- NDATA - Number of sets of data.
- NOBS - Number of observations in data set.
- NX - Number of independent variables for each observation.
- TOLER - Tolerance input. When the maximum fractional change in the parameter estimates is less than or equal to this tolerance, the iterative procedure will stop and the program will continue to the next equation.
- X(I,J) - Value of the J^{th} independent variable for the I^{th} observation.
- Y(I) - Value of the dependent variable for the I^{th} observation.
- NEQUAT - Number of the equation to be fitted.
- NNX(1), · · · NNX(7) - The independent variable numbers, to be used in the chosen equation, in order of use.
- THETA(K) - The initial estimates of the parameters for the chosen equation.

Two additional equations can be inserted into the SUBROUTINE FUNCT. To fit these equations set the equation number equal to 6 or 7 and insert a card with NTHETA in Columns 1-5 under I 5 format immediately following the card declaring the value of NEQUAT and preceding the card declaring NNX (1), · · · NNX (7). If the equation number equals 6, insert the equation followed by its derivatives with respect to Θ_1, Θ_2, · · · Θ_m immediately following statement 106 in SUBROUTINE FUNCT. If

the equation number equals 7, insert the equation followed by its derivatives with respect to Θ_1, Θ_2, · · · Θ_m immediately following statement 107 in SUBROUTINE FUNCT. The expressions for the equation and its derivatives must be in FORTRAN V computer language. The following names should be applied as needed.

FZ (I) - The dependent variable for the I^{th} observation estimated from the equation.

THETA (K) - The K^{th} parameter, Θ_k.

X(I,J) - The J^{th} independent variable for the I^{th} observation.

DFZ (I,K) - The derivative of the equation with respect to Θ_k evaluated for the I^{th} observation.

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2.3 SAMPLE OUTPUTS

```
DATA SET 1

EQUATION     2

OBSERVATIONS
NO.          X(1)            X(2)            Y
 1          2.00000         4.50000         7.50000
 2          1.30000         4.15000         5.90000
 3          1.00000         3.30000        13.10000
 4           .90000         3.25000        14.10000
 5           .70000         4.45000        19.10000
 6          1.90000         3.55000         4.96000


NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED= 1


INITIAL ESTIMATES OF THETA
THETA(1)=       10.0000000000
THETA(2)=       -1.0000000000
```


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FINAL ESTIMATES OF THETA
THETA(1)= 12.1527241468
THETA(2)= -1.2609184831

NO.	Y	Y-HAT
1	7.50000	5.07107
2	5.90000	8.72972
3	13.10000	12.15272
4	14.10000	13.87938
5	19.10000	19.05427
6	4.96000	5.40989

	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-RATIO
TOTAL	5	154.07235		
REGRESSION	2	139.01483	69.50741	13.848
ERROR	3	15.05753	5.01918	

COEFFICIENT OF DETERMINATION (R-SQUARE) = .90227

STANDARD ERROR OF THE ESTIMATE = 2.24035

Data Source:
Headquarters Armament Development and Test Center, Cost Analysis Division. Air Launched Weapon Systems Cost Model, Volume II (U). Eglin Air Force Base, Fla., January 1974, p. 5. (Publication SECRET.)

DATA SET 2

EQUATION 3

OBSERVATIONS

NO.	X(1)	X(2)	X(3)	Y
1	20.00000	11.00000	30.00000	43.30000
2	224.00000	.73000	48.00000	52.70000
3	7.20000	17.00000	36.00000	28.50000
4	10.20000	29.90000	96.00000	93.70000
5	10.00000	88.20000	96.00000	104.20000
6	62.00000	3.60000	51.00000	53.10000
7	60.00000	.56000	30.00000	20.40000
8	129.00000	.55000	51.00000	26.50000
9	170.00000	.45000	65.00000	38.00000
10	932.00000	.01000	22.00000	7.90000

NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED= 1 2

INITIAL ESTIMATES OF THETA
THETA(1)= .2500000000
THETA(2)= 1.0000000000
THETA(3)= 1.0000000000

FINAL ESTIMATES OF THETA

THETA(1)=	3.5679777861
THETA(2)=	.5100684389
THETA(3)=	.5045985505

NO.	Y	Y-HAT
1	43.30000	55.14698
2	52.70000	48.11078
3	28.50000	40.79464
4	93.70000	64.78863
5	104.20000	110.70601
6	53.10000	55.89513
7	20.40000	21.49498
8	26.50000	31.47420
9	38.00000	32.74276
10	7.90000	11.42432

	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-RATIO
TOTAL	9	8615.70170		
REGRESSION	3	7351.12231	2450.37410	11.626
ERROR	6	1264.57939	210.76323	

COEFFICIENT OF DETERMINATION (R-SQUARE) = .85322

STANDARD ERROR OF THE ESTIMATE = 14.51769

Data Source:
Brad C. Frederic. Parametric Cost Estimating Relationships for Missile Preproduction and Production-Support Elements, Phase I Progress Report. Tecolote Research Incorporated. Santa Barbara, Calif., August 1974. Table 7.

DATA SET 3

EQUATION 2

OBSERVATIONS

NO.	X(1)	Y
1	250.00000	.71100
2	1000.00000	1.41700
3	152.00000	.54100
4	143.00000	.62200
5	25.00000	.20400
6	73.00000	.49500
7	69.00000	.63200
8	250.00000	.71500

NUMEER(S) OF THE INDEPENDENT VARIABLE(S) FITTED= 1

INITIAL ESTIMATES OF THETA
THETA(1)= .0999999996
THETA(2)= .5000000000

32

FINAL ESTIMATES OF THETA

THETA(1)= .0724704023

THETA(2)= .4275295362

NO.	Y	Y-HAT
1	.71100	.76798
2	1.41700	1.38915
3	.54100	.62082
4	.62200	.60483
5	.20400	.28696
6	.49500	.45372
7	.63200	.44292
8	.71500	.76798

	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-RATIO
TOTAL	7	.82982		
REGRESSION	2	.77199	.38599	33.371
ERROR	5	.05783	.01157	

COEFFICIENT OF DETERMINATION (R-SQUARE) = .93031

STANDARD ERROR OF THE ESTIMATE = .10755

Data Source:
This data was collected from several sources. The regression equation is presented in the following document: Naval Weapons Center. An Independent Cost Analysis of the HARM Weapon System RDT&E (U), by Douglas R. Johnson, Dr. Barbara J. Bell, Robert E. Sax, Paul A. Douillard and W. Eugene Waller. China Lake, Calif., NWC, October 1975, p. 38. (Publication CONFIDENTIAL.)

DATA SET 4

EQUATION 2

OBSERVATIONS

NO.	X(1)	Y
1	74500.00000	22840.00000
2	74600.00000	15238.00000
3	59875.00000	12390.00000
4	6540.00000	4142.00000
5	4063.00000	1516.00000
6	3694.00000	1321.00000
7	97120.00000	15936.00000
8	97120.00000	17975.00000
9	21300.00000	5188.00000
10	14100.00000	3219.00000
11	23370.00000	5567.00000
12	21300.00000	5188.00000
13	19820.00000	8523.00000
14	129.00000	127.00000
15	1170.00000	621.00000
16	1947.00000	700.00000
17	23590.00000	2739.00000
18	54150.00000	9296.00000
19	51200.00000	5842.00000
20	6060.00000	3010.00000
21	7050.00000	2367.00000
22	58300.00000	8543.00000
23	7973.00000	7056.00000
24	61800.00000	9019.00000
25	21648.00000	4656.00000
26	19760.00000	4803.00000
27	13468.00000	2233.00000

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```
NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED= 1

INITIAL ESTIMATES OF THETA
THETA(1)=     2.0000000000
THETA(2)=     1.0000000000
```

```
FINAL ESTIMATES OF THETA
THETA(1)=         1.6965622306
THETA(2)=          .8045727462
```

NO.	Y	Y-HAT
1	22840.00000	14111.52490
2	15238.00000	14126.76283
3	12390.00000	11836.17415
4	4142.00000	1992.87318
5	1516.00000	1358.78140
6	1321.00000	1258.57924
7	15936.00000	17467.16529
8	17975.00000	17467.16529
9	5188.00000	5153.07019
10	3219.00000	3697.58708
11	5567.00000	5552.30777
12	5188.00000	5153.07019
13	8523.00000	4862.97771
14	127.00000	84.66282
15	621.00000	499.05512
16	700.00000	751.80289
17	2739.00000	5594.32280
18	9296.00000	10916.76866
19	5842.00000	10435.66332
20	3010.00000	1874.32191
21	2367.00000	2116.98548
22	8543.00000	11585.02114
23	7056.00000	2337.26702
24	9019.00000	12141.39389
25	4656.00000	5220.70054
26	4803.00000	4851.12974
27	2233.00000	3563.64583

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	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F-RATIO
TOTAL	26	870169787.62963		
REGRESSION	2	694950131.57309	347475065.78654	47.594
ERROR	24	175219656.05654	7300819.00236	

COEFFICIENT OF DETERMINATION (R-SQUARE) = .79864

STANDARD ERROR OF THE ESTIMATE = 2702.00278

Data Source:
Naval Weapons Center. Solid Motor Cost and Reliability Prediction Methodology, Final Technical Report (U), by Booz-Allen Applied Research. China Lake, Calif., NWC, March 1974. Tables 5 and 6. (NWC TP 5634, Publication CONFIDENTIAL.)

3.0 HEWLETT-PACKARD 9830A CALCULATOR PRESENTATION

3.1 PROGRAM *

```
10 REM DR. BARBARA J. BELL   NONLINEAR REGRESSION
20 DIM AS[7,8],FS[30],DS[30,7],BS[7],ES[7],GS[7],HS[7],LS[7,8],MS[7,8],OS[7]
30 DIM PS[8],XS[30,7],TS[7],KS[7],CS[8],YS[30],WS[8],US[7,8],VS[7]
40 DISP "FULL PRINT? 1=YES, 0=NO";
50 INPUT N6
60 DISP "ENTER NO. OF OBSERVATIONS";
70 INPUT N1
80 DISP "ENTER NO. OF IND. VAR. (X'S)";
90 INPUT N2
100 DISP "ENTER TOLERANCE (.01)";
110 INPUT T1
120 FOR I=1 TO N1
130 DISP "INPUT Y("I")";
140 INPUT Y[I]
150 FOR J=1 TO N2
160 DISP "INPUT X("I","J")";
170 INPUT X[I,J]
180 NEXT J
190 NEXT I
200 PRINT
210 PRINT
220 DISP "EQUATION NO.";
230 INPUT N4
240 N5=0
250 IF N4>2 THEN 300
260 N3=2
270 GOSUB 2630
280 L3=1
290 GOTO 520
300 IF N4>3 THEN 360
310 N3=3
320 GOSUB 2630
330 GOSUB 2670
340 L3=2
350 GOTO 520
360 IF N4>5 THEN 440
370 N3=4
380 GOSUB 2630
390 GOSUB 2670
400 GOSUB 2710
```

* **The core required by the main and KEY programs is 3801 words (16 bits-2 bytes per word). The additional core required to execute the program varies with the number of observations and the number of independent variables fitted. For example, the additional core required to execute Data Set 1 is 1933 words and for Data Set 2 is 1951 words.**

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```
410 L3=3
420 GOTO 520
430 DISP "ENTER NO. OF THETAS";
440 GOSUB 2630
450 GOSUB 2670
460 GOSUB 2710
470 GOSUB 2750
480 GOSUB 2790
490 GOSUB 2830
500 GOSUB 2870
510 L3=7
520 N7=N3+1
530 FOR K1=1 TO N3
540 DISP "ENTER THETA ESTIMATE"K1;
550 INPUT T[K1]
560 NEXT K1
570 PRINT
580 PRINT "EQUATION "N4
590 PRINT
600 PRINT "OBSERVATIONS"
610 PRINT " NO.       Y          ";
620 T9=-INT(-N2/4)
630 FOR I=0 TO T9-1
640 P9=4*(I<T9-1)+(N2-4*(T9-1))*(I=T9-1)
650 FOR J=1 TO P9
660 S9=4*I+J
670 WRITE (15,680)"X("S9")      ",
680 FORMAT 4F2.0
690 NEXT J
700 PRINT
710 IF I=T9-1 THEN 730
720 PRINT TAB19;
730 NEXT I
740 FOR I=1 TO N1
750 P1=0
760 WRITE (15,770)I,Y[I];
770 FORMAT F3.0,F11.3
780 FOR J=1 TO N2
790 P1=P1+1
800 IF ((P1=4)+(J=N2))=0 THEN 890
810 FOR N=J-P1+1 TO J
820 WRITE (15,830)X[I,N];
830 FORMAT 4F10.3
840 NEXT N
850 PRINT
860 IF J=N2 THEN 890
870 PRINT TAB14;
880 P1=0
890 NEXT J
900 NEXT I
```

```
910 PRINT
920 PRINT "NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED =";
930 FOR J=1 TO L3
940 WRITE (15,950)V[J];
950 FORMAT 7F2.0
960 NEXT J
970 PRINT
980 PRINT
990 PRINT "INITIAL ESTIMATES OF THETA"
1000 S1=1
1010 GOSUB 2920
1020 P1=FNQ1
1030 Q0=0
1040 T2=0
1050 S1=0
1060 FOR I=1 TO N1
1070 T2=T2+Y[I]↑2
1080 S1=S1+Y[I]
1090 Q0=Q0+(Y[I]-F[I])↑2
1100 NEXT I
1110 C1=S1↑2/N1
1120 T2=T2-C1
1130 M4=N1-1
1140 M3=N1-N3-1
1150 N5=N5+1
1160 R1=T2-Q0
1170 R2=R1/T2
1180 R3=R1/N3
1190 E1=Q0/M3
1200 IF Q0 <= 0 THEN 1230
1210 F1=R3/E1
1220 S1=E1↑0.5
1230 IF N5 >= 15 THEN 2530
1240 IF N5=1 THEN 1260
1250 IF A2 <= T1 THEN 2530
1260 IF N6=0 THEN 1280
1270 GOSUB 3080
1280 FOR J=1 TO N3
1290 FOR K1=1 TO N3
1300 A1=0
1310 FOR I=1 TO N1
1320 A1=A1+D[I,K1]*D[I,J]
1330 NEXT I
1340 IF N6=0 THEN 1360
1350 PRINT "A$ ="A1
1360 A[K1,J]=A1
1370 NEXT K1
1380 NEXT J
1390 FOR K1=1 TO N3
1400 B1=0
```


40

```
1410 FOR I=1 TO N1
1420 B1=B1+(Y[I]-F[I])*D[I,K1]
1430 NEXT I
1440 B[K1]=B1
1450 NEXT K1
1460 IF N6=0 THEN 1530
1470 FOR K1=1 TO N3
1480 PRINT "B("K1")="B[K1]
1490 NEXT K1
1500 FOR K1=1 TO N3
1510 A[K1,N7]=B[K1]
1520 NEXT K1
1530 FOR K1=1 TO N3
1540 K[K1]=K1
1550 NEXT K1
1560 FOR J=1 TO N7
1570 C[J]=J
1580 NEXT J
1590 I1=0
1600 N=N3
1610 P1=FNR1
1620 I1=I1+1
1630 P1=FNS1
1640 IF N3 <= I1 THEN 1680
1650 N=N-1
1660 IF N <= 1 THEN 1620
1670 GOTO 1610
1680 P1=FNP1
1690 FOR K1=1 TO N3
1700 E[K1]=A[K1,1]
1710 NEXT K1
1720 IF N6=0 THEN 1760
1730 FOR K1=1 TO N3
1740 PRINT "D("K1")="E[K1]
1750 NEXT K1
1760 FOR K1=1 TO N3
1770 G[K1]=E[K1]/T[K1]
1780 H[K1]=ABS(G[K1])
1790 NEXT K1
1800 IF N6=0 THEN 1840
1810 FOR K1=1 TO N3
1820 PRINT "DD("K1")="H[K1]
1830 NEXT K1
1840 IF N3>1 THEN 1870
1850 A2=H[1]
1860 GOTO 1970
1870 K1=2
1880 I=1
1890 IF H[I] >= H[K1] THEN 1910
1900 I=K1
```


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```
1910 IF N3 <= K1 THEN 1940
1920 K1=K1+1
1930 GOTO 1890
1940 A2=H[I]
1950 IF N6=0 THEN 1970
1960 PRINT "AMXD ="A2
1970 IF A2 <= T1 THEN 1160
1980 FOR K1=1 TO N3
1990 T[K1]=T[K1]+0.5*E[K1]
2000 NEXT K1
2010 P1=FNQ1
2020 IF N6=0 THEN 2040
2030 GOSUB 2910
2040 Q5=0
2050 FOR I=1 TO N1
2060 Q5=Q5+(Y[I]-F[I])↑2
2070 NEXT I
2080 IF N6=0 THEN 2100
2090 PRINT "Q5 ="Q5
2100 IF Q5<Q0 THEN 2470
2110 FOR K1=1 TO N3
2120 T[K1]=T[K1]+0.5*E[K1]
2130 NEXT K1
2140 P1=FNQ1
2150 IF N6=0 THEN 2170
2160 GOSUB 2910
2170 Q1=0
2180 FOR I=1 TO N1
2190 Q1=Q1+(Y[I]-F[I])↑2
2200 NEXT I
2210 IF N6=0 THEN 2230
2220 PRINT "Q1 ="Q1
2230 T3=0.5+0.25*(Q0-Q1)/(Q0+Q1-2*Q5)
2240 IF N6=0 THEN 2260
2250 PRINT "TMIN ="T3
2260 IF T3<0 THEN 2290
2270 IF T3=0 THEN 2530
2280 IF T3 <= 1 THEN 2310
2290 T3=1
2300 GOTO 2440
2310 FOR K1=1 TO N3
2320 T[K1]=T[K1]-(1-T3)*E[K1]
2330 NEXT K1
2340 P1=FNQ1
2350 IF N6=0 THEN 2370
2360 GOSUB 2910
2370 Q1=0
2380 FOR I=1 TO N1
2390 Q1=Q1+(Y[I]-F[I])↑2
2400 NEXT I
```


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```
2410 IF N6=0 THEN 2430
2420 PRINT "QTMIN ="Q1
2430 IF Q0>Q1 THEN 2500
2440 FOR K1=1 TO N3
2450 T[K1]=T[K1]+(0.5-T3)*E[K1]
2460 NEXT K1
2470 Q0=Q5
2480 IF N5 >= 15 THEN 1160
2490 GOTO 1150
2500 Q0=Q1
2510 IF N5 >= 15 THEN 1160
2520 GOTO 1150
2530 PRINT
2540 PRINT
2550 PRINT "FINAL ESTIMATES OF THETA"
2560 GOSUB 2920
2570 GOSUB 3080
2580 WRITE (15,3270)
2590 DISP "ANOTHER EQUATION? 0=NO, 1=YES";
2600 INPUT Z9
2610 IF Z9#0 THEN 200
2620 STOP
2630 DISP "ENTER J FOR FIRST X(I,J)";
2640 INPUT J1
2650 V[1]=J1
2660 RETURN
2670 DISP "ENTER J FOR SECOND X(I,J)";
2680 INPUT J2
2690 V[2]=J2
2700 RETURN
2710 DISP "ENTER J FOR THIRD X(I,J)";
2720 INPUT J3
2730 V[3]=J3
2740 RETURN
2750 DISP "ENTER J FOR 4TH X(I,J)";
2760 INPUT J4
2770 V[4]=J4
2780 RETURN
2790 DISP "ENTER J FOR 5TH X(I,J)";
2800 INPUT J5
2810 V[5]=J5
2820 RETURN
2830 DISP "ENTER J FOR 6TH X(I,J)";
2840 INPUT J6
2850 V[6]=J6
2860 RETURN
2870 DISP "ENTER J FOR 7TH X(I,J)";
2880 INPUT J7
2890 V[7]=J7
2900 RETURN
```

```
2910 PRINT
2920 FOR K1=1 TO N3
2930 WRITE (15,2940)"THETA(",K1,") ="T[K1]
2940 FORMAT F2.0,F20.10
2950 NEXT K1
2960 IF S1=1 THEN 3070
2970 PRINT
2980 PRINT " NO.            Y                  Y-HAT"
2990 FOR N=1 TO N1
3000 WRITE (15,3010)N,Y[N],F[N]
3010 FORMAT F3.0," ",2F15.5
3020 IF N6=0 THEN 3060
3030 FOR N9=1 TO N3
3040 PRINT "DERIVATIVE("N9") ="D[N,N9]
3050 NEXT N9
3060 NEXT N
3070 RETURN
3080 PRINT
3090 PRINT "TOTAL DEGREES OF FREEDOM        ="M4
3100 PRINT "REGRESSION DEGREES OF FREEDOM ="N3
3110 PRINT "ERROR DEGREES OF FREEDOM        ="M3
3120 PRINT
3130 PRINT "TOTAL SUM OF SQUARES        ="T2
3140 PRINT "REGRESSION SUM OF SQUARES ="R1
3150 IF E1 <= 0 THEN 3260
3160 PRINT "ERROR SUM OF SQUARES        ="Q0
3170 PRINT
3180 PRINT "REGRESSION MEAN SQUARE ="R3
3190 PRINT "ERROR MEAN SQUARE       ="E1
3200 PRINT
3210 PRINT "F-RATIO ="F1
3220 PRINT
3230 PRINT "COEFFICIENT OF DETERMINATION (R-SQUARE) ="R2
3240 PRINT
3250 PRINT "STANDARD ERROR OF THE ESTIMATE ="S1
3260 WRITE (15,3270)
3270 FORMAT /,/,/,/
3280 RETURN
3290 END
```

KEY f_o

```
2000 DEF FNQ(P1)
2010 IF N4>1 THEN 2080
2020 FOR I=1 TO N1
2030 F[I]=T[1]*EXP(T[2]*X[I,J1])
2040 D[I,1]=EXP(T[2]*X[I,J1])
2050 D[I,2]=T[1]*EXP(T[2]*X[I,J1])*X[I,J1]
2060 NEXT I
2070 GOTO 2450
2080 IF N4>2 THEN 2150
2090 FOR I=1 TO N1
2100 F[I]=T[1]*X[I,J1]↑T[2]
2110 D[I,1]=X[I,J1]↑T[2]
2120 D[I,2]=T[1]*X[I,J1]↑T[2]*LOG(X[I,J1])
2130 NEXT I
2140 GOTO 2450
2150 IF N4>3 THEN 2230
2160 FOR I=1 TO N1
2170 F[I]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]
2180 D[I,1]=X[I,J1]↑T[2]*X[I,J2]↑T[3]
2190 D[I,2]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*LOG(X[I,J1])
2200 D[I,3]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*LOG(X[I,J2])
2210 NEXT I
2220 GOTO 2450
2230 IF N4>4 THEN 2320
2240 FOR I=1 TO N1
2250 F[I]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*X[I,J3]↑T[4]
2260 D[I,1]=X[I,J1]↑T[2]*X[I,J2]↑T[3]*X[I,J3]↑T[4]
2270 D[I,2]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*X[I,J3]↑T[4]*LOGX[I,J1]
2280 D[I,3]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*X[I,J3]↑T[4]*LOGX[I,J2]
2290 D[I,4]=T[1]*X[I,J1]↑T[2]*X[I,J2]↑T[3]*X[I,J3]↑T[4]*LOGX[I,J3]
2300 NEXT I
2310 GOTO 2450
2320 IF N4>5 THEN 2410
2330 FOR I=1 TO N1
2340 F[I]=T[1]*X[I,J1]↑T[2]+T[3]*X[I,J2]↑T[4]
2350 D[I,1]=X[I,J1]↑T[2]
2360 D[I,2]=T[1]*X[I,J1]↑T[2]*LOGX[I,J1]
2370 D[I,3]=X[I,J2]↑T[4]
2380 D[I,4]=T[3]*X[I,J2]↑T[4]*LOGX[I,J2]
2390 NEXT I
2400 GOTO 2450
2410 IF N4>6 THEN 2440
2420 REM: ADD EQUATION (N4=6) HERE
2430 GOTO 2450
2440 REM: ADD EQUATION (N4=7) HERE
2450 RETURN 0
2460 END
```

KEY f_1

```
3000 DEF FNR(P1)
3010 FOR I=1 TO N3
3020 FOR J=1 TO N7
3030 L[I,J]=ABS(A[I,J])
3040 NEXT J
3050 NEXT I
3060 K1=I=J=1
3070 L1=2
3080 IF L[I,J] >= L[K1,L1] THEN 3110
3090 I=K1
3100 J=L1
3110 IF N<L1 THEN 3150
3120 IF N>L1 THEN 3230
3130 IF N=K1 THEN 3220
3140 IF N>K1 THEN 3170
3150 PRINT "N ="N"N < L OR K"
3160 GOTO 3560
3170 K1=K1+1
3180 L1=1
3190 GOTO 3080
3200 L1=L1+1
3210 GOTO 3080
3220 K2=I
3230 L2=J
3240 FOR I=1 TO N3
3250 FOR J=1 TO N7
3260 M[I,J]=A[I,J]
3270 NEXT J
3280 NEXT I
3290 FOR J=1 TO N7
3300 A[1,J]=M[K2,J]
3310 A[K2,J]=M[1,J]
3320 NEXT J
3330 FOR I=1 TO N3
3340 FOR J=1 TO N7
3350 M[I,J]=A[I,J]
3360 NEXT J
3370 NEXT I
3380 FOR I=1 TO N3
3390 A[I,1]=M[I,L2]
3400 A[I,L2]=M[I,1]
3410 NEXT I
3420 I3=1+I1
3430 I2=K2+I1
```


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```
3440 FOR I=1 TO N3
3450 O[I]=K[I]
3460 NEXT I
3470 K[I3]=O[I2]
3480 K[I2]=O[I3]
3490 Z1=1+I1
3500 Z2=L2+I1
3510 FOR J=1 TO N7
3520 P[J]=C[J]
3530 NEXT J
3540 C[Z1]=P[Z2]
3550 C[Z2]=P[Z1]
3560 RETURN 0
3570 END
```

KEY f_2

```
4000 DEF FNS(P1)
4010 FOR J=1 TO N3
4020 W[J]=A[1,J+1]/A[1,1]
4030 NEXT J
4040 W[N7]=1/A[1,1]
4050 N8=N3-1
4060 FOR I=1 TO N8
4070 FOR J=1 TO N3
4080 A[I,J]=A[I+1,J+1]-A[I+1,1]*W[J]
4090 NEXT J
4100 A[I,N7]=0-A[I+1,1]*W[N7]
4110 NEXT I
4120 FOR J=1 TO N7
4130 A[N3,J]=W[J]
4140 NEXT J
4150 RETURN 0
4160 END
```

KEY f_3

```
5000 DEF FNP(P1)
5010 FOR I=1 TO N3
5020 FOR J=1 TO N7
5030 U[I,J]=A[I,J]
5040 NEXT J
5050 NEXT I
5060 FOR I=1 TO N3
5070 M1=C[I]
5080 FOR J=1 TO N7
5090 A[M1,J]=U[I,J]
5100 NEXT J
5110 NEXT I
5120 FOR I=1 TO N3
5130 FOR J=1 TO N7
5140 U[I,J]=A[I,J]
5150 NEXT J
5160 NEXT I
5170 FOR J=1 TO N3
5180 M2=K[J]
5190 FOR I=1 TO N3
5200 A[I,M2+1]=U[I,J+1]
5210 NEXT I
5220 NEXT J
5230 RETURN 0
5240 END
```

3.2 INPUTS

1. Insert cassette.
2. Key in: LOAD (N), where N = file number of main program.
3. EXECUTE
4. Key in: LOAD KEY (M), where M = file number of KEY programs.
5. EXECUTE
6. RUN
7. EXECUTE
8. "FULL PRINT? 1 = YES, 0 = NO" will be displayed.
9. Key in: 0 for print-out of final iteration only.
   1 for full print-out.
10. EXECUTE
11. "ENTER NO. OF OBSERVATIONS" will be displayed.
12. Key in: Number of observations to be inputted.
13. EXECUTE
14. "ENTER NO. OF IND. VAR. (X'S)" will be displayed.
15. Key in: Number of independent variables to be inputted.
16. EXECUTE
17. "ENTER TOLERANCE (.01)" will be displayed.
18. Key in: Tolerance input. When the maximum fractional change in the parameter estimate(s) is less than or equal to this tolerance, the iterative procedure will stop and the program will continue to the next equation.
19. EXECUTE
20. "INPUT Y(I)" will be displayed.
21. Key in: Value of the dependent variable for the I^{th} observation.
22. EXECUTE
23. "INPUT X(I, J)" will be displayed.
24. Key in: Value of the J^{th} independent variable for the I^{th} observation.
25. EXECUTE
    Steps 23 - 25 will be repeated for all X variables for the I^{th} observation.
26. Steps 20 - 25 will be repeated for all observations.
27. "EQUATION NO." will be displayed.
28. Key in: Number of the equation to be fitted.
29. EXECUTE
30. "ENTER J FOR FIRST X(I, J)" will be displayed.
31. Key in: The first independent variable number to be used in the chosen equation.
32. EXECUTE
33. For equations 1-4 steps 30 - 32 will be repeated up to the number of independent variable(s) used in the chosen equation. For equation 5 enter in order the numbers of the two independent variables to be used. When the number of a third independent variable is requested, enter 0. (Only two independent variables

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are required for equation 5. The input of 0 as the third independent variable is for the purpose of saving calculator storage space.) If user equations are to be fitted, steps 30 - 32 will be repeated to allow the user to indicate up to 7 independent variables. When the ordered list of the independent variable numbers is completed, enter 0 when step 30 is displayed.

34. "ENTER THETA ESTIMATE K" will be displayed.
35. Key in: The initial estimate of the K^{th} parameter, Θ_k.
36. EXECUTE
37. For equations 1-5 steps 34 - 36 will be repeated up to the number of parameters for the chosen equation. If user equations are to be fitted, steps 34 - 36 will be repeated to allow the user to input up to 7 parameter estimates. When the ordered list of parameter estimates is completed, enter 0 each time step 34 is displayed.
38. An equation is fitted until the tolerance level is reached or until 15 iterations have been completed.
39. "ANOTHER EQUATION? 0 = NO, 1 = YES" will be displayed.
40. Key in: 0 or 1 as indicated. If 1 is keyed in, the program will return to step 27.
41. When all desired equations have been estimated for the data set, another data set can be entered by pressing the RUN, EXECUTE keys.

Two additional equations can be inserted into the first KEY program (KEY f_o). To fit these equations set the equation number equal to 6 or 7 at step 28. Before step 30 "ENTER NO. OF THETAS" will be displayed. Key in: The number of parameters, Θ's, in the equation to be inserted. Press EXECUTE. Continue at step 30. If equation number equals 6, insert the equation followed by its derivatives with respect to $\Theta_1, \Theta_2, \cdots \Theta_m$ immediately following statement 2420 in KEY f_o. If equation number equals 7, insert the equation followed by its derivatives with respect to $\Theta_1, \Theta_2, \cdots \Theta_m$ immediately following statement 2440 in KEY f_o. The expressions for the equation and its derivatives must be in BASIC computer language. The following names should be applied as needed.

F(I)	- The dependent variable for the I^{th} observation estimated from the equation.
T(K)	- The K^{th} parameter, Θ_k.
X(I,J)	- The J^{th} independent variable for the I^{th} observation.
D(I,K)	- Derivative of the equation with respect to Θ_k evaluated for the I^{th} observation.

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3.3 SAMPLE OUTPUTS

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DATA SET 1

```
EQUATION  2

OBSERVATIONS
 NO.       Y            X( 1)       X( 2)
  1       7.500         2.000       4.500
  2       5.900         1.300       4.150
  3      13.100         1.000       3.300
  4      14.100         0.900       3.250
  5      19.100         0.700       4.450
  6       4.960         1.900       3.550

NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED = 1

INITIAL ESTIMATES OF THETA
THETA( 1) =          10.0000000000
THETA( 2) =          -1.0000000000

FINAL ESTIMATES OF THETA
THETA( 1) =          12.1526000000
THETA( 2) =          -1.2609100000

  NO.             Y                 Y-HAT
   1           7.50000             5.07104
   2           5.90000             8.72965
   3          13.10000            12.15260
   4          14.10000            13.87920
   5          19.10000            19.05400
   6           4.96000             5.40985
```

Data Source:

Headquarters Armament Development and Test Center, Cost Analysis Division. Air Launched Weapon Systems Cost Model, Volume II (U). Eglin Air Force Base, Fla., January 1974, p. 5. (Publication SECRET.)

```
TOTAL DEGREES OF FREEDOM       = 5
REGRESSION DEGREES OF FREEDOM = 2
ERROR DEGREES OF FREEDOM       = 3

TOTAL SUM OF SQUARES       = 154.0723333
REGRESSION SUM OF SQJARES = 139.0147671
ERROR SUM OF SQUARES       = 15.05756623

REGRESSION MEAN SQUARE = 69.50738355
ERROR MEAN SQUARE       = 5.019188742

F-RATIO = 13.8483303

COEFFICIENT OF DETERMINATION (R-SQUARE) = 0.902269500

STANDARD ERROR OF THE ESTIMATE = 2.240354602
```

DATA SET 2

EQUATION 3

OBSERVATIONS

NO.	Y	X(1)	X(2)	X(3)
1	43.300	20.000	11.000	30.000
2	52.700	224.000	0.730	48.000
3	28.500	7.200	17.000	36.000
4	93.700	10.200	29.900	96.000
5	104.200	10.000	88.200	96.000
6	53.100	62.000	3.600	51.000
7	20.400	60.000	0.560	30.000
8	26.500	129.000	0.550	51.000
9	38.000	170.000	0.450	65.000
10	7.900	932.000	0.010	22.000

NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED = 1 2

INITIAL ESTIMATES OF THETA
THETA(1) = 0.2500000000
THETA(2) = 1.0000000000
THETA(3) = 1.0000000000

FINAL ESTIMATES OF THETA
THETA(1) = 3.5661600000
THETA(2) = 0.5100470000
THETA(3) = 0.5046260000

NO.	Y	Y-HAT
1	43.30000	55.11890
2	52.70000	48.08020
3	28.50000	40.77530
4	93.70000	64.75840
5	104.20000	110.65700
6	53.10000	55.86360
7	20.40000	21.48170
8	26.50000	31.45430
9	38.00000	32.72170
10	7.90000	11.41530

Data Source:

Brad C. Frederic. Parametric Cost Estimating Relationships for Missile Preproduction and Production-Support Elements, Phase I Progress Report. Tecolote Research Incorporated. Santa Barbara, Calif., August 1974. Table 7.

```
TOTAL DEGREES OF FREEDOM       = 9
REGRESSION DEGREES OF FREEDOM = 3
ERROR DEGREES OF FREEDOM       = 6

TOTAL SUM OF SQUARES        = 8615.701
REGRESSION SUM OF SQUARES = 7351.109568
ERROR SUM OF SQUARES        = 1264.591432

REGRESSION MEAN SQUARE = 2450.369856
ERROR MEAN SQUARE        = 210.7652387

F-RATIO = 11.62606259

COEFFICIENT OF DETERMINATION (R-SQUARE) = 0.853222456

STANDARD ERROR OF THE ESTIMATE = 14.51775598
```


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DATA SET 3

EQUATION 2

OBSERVATIONS

NO.	Y	X(1)
1	0.711	250.000
2	1.417	1000.000
3	0.541	152.000
4	0.622	143.000
5	0.204	25.000
6	0.495	73.000
7	0.632	69.000
8	0.715	250.000

NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED = 1

INITIAL ESTIMATES OF THETA
THETA(1) = 0.1000000000
THETA(2) = 0.5000000000

FINAL ESTIMATES OF THETA
THETA(1) = 0.0724703000
THETA(2) = 0.4275290000

NO.	Y	Y-HAT
1	0.71100	0.76798
2	1.41700	1.38914
3	0.54100	0.62081
4	0.62200	0.60482
5	0.20400	0.28696
6	0.49500	0.45372
7	0.63200	0.44292
8	0.71500	0.76798

Data Source:
This data was collected from several sources. The regression equation is presented in the following document:

Naval Weapons Center. <u>An Independent Cost Analysis of the HARM Weapon System RDT&E</u> (U), by Douglas R. Johnson, Dr. Barbara J. Bell, Robert E. Sax, Paul A. Douillard, and W. Eugene Waller. China Lake, Calif., NWC, October 1975, p. 38. (Publication CONFIDENTIAL.)

```
TOTAL DEGREES OF FREEDOM        = 7
REGRESSION DEGREES OF FREEDOM = 2
ERROR DEGREES OF FREEDOM        = 5

TOTAL SUM OF SQUARES        = 0.829818875
REGRESSION SUM OF SQUARES = 0.771985047
ERROR SUM OF SQUARES        = 0.057833828

REGRESSION MEAN SQUARE = 0.385992523
ERROR MEAN SQUARE        = 0.011566766

F-RATIO = 33.37082595

COEFFICIENT OF DETERMINATION (R-SQUARE) = 0.930305480

STANDARD ERROR OF THE ESTIMATE = 0.107548899
```


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DATA SET 4

EQUATION 2

OBSERVATIONS

NO.	Y	X(1)
1	22840.000	74500.000
2	15238.000	74600.000
3	12390.000	59875.000
4	4142.000	6540.000
5	1516.000	4063.000
6	1321.000	3694.000
7	15936.000	97120.000
8	17975.000	97120.000
9	5188.000	21300.000
10	3219.000	14100.000
11	5567.000	23370.000
12	5188.000	21300.000
13	8523.000	19820.000
14	127.000	129.000
15	621.000	1170.000
16	700.000	1947.000
17	2739.000	23590.000
18	9296.000	54150.000
19	5842.000	51200.000
20	3010.000	6060.000
21	2367.000	7050.000
22	8543.000	58300.000
23	7056.000	7973.000
24	9019.000	61800.000
25	4656.000	21648.000
26	4803.000	19760.000
27	2233.000	13468.000

NUMBER(S) OF THE INDEPENDENT VARIABLE(S) FITTED = 1

INITIAL ESTIMATES OF THETA
THETA(1) = 2.0000000000
THETA(2) = 1.0000000000

FINAL ESTIMATES OF THETA
THETA(1) = 1.6891500000
THETA(2) = 0.8049880000

Data Source:

Naval Weapons Center. Solid Motor Cost and Reliability Prediction Methodology, Final Technical Report (U), by Booz-Allen Applied Research. China Lake, Calif., NWC, March 1974. Tables 5 and 6. (NWC TP 5634, Publication CONFIDENTIAL.)

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NO.	Y	Y-HAT
1	22640.00000	14115.40000
2	15238.00000	14130.70000
3	12390.00000	11838.40000
4	4142.00000	1991.41000
5	1516.00000	1357.52000
6	1321.00000	1257.36000
7	15936.00000	17473.90000
8	17975.00000	17473.90000
9	5188.00000	5151.83000
10	3219.00000	3696.06000
11	5567.00000	5551.18000
12	5188.00000	5151.83000
13	8523.00000	4861.66000
14	127.00000	84.46320
15	621.00000	498.33400
16	700.00000	750.87600
17	2739.00000	5593.21000
18	9296.00000	10918.30000
19	5842.00000	10436.90000
20	3010.00000	1872.89000
21	2367.00000	2115.50000
22	8543.00000	11587.00000
23	7056.00000	2335.75000
24	9019.00000	12143.80000
25	4656.00000	5219.48000
26	4803.00000	4849.81000
27	2233.00000	3562.11000

TOTAL DEGREES OF FREEDOM = 26
REGRESSION DEGREES OF FREEDOM = 2
ERROR DEGREES OF FREEDOM = 24

TOTAL SUM OF SQUARES = 870169787.6
REGRESSION SUM OF SQUARES = 694949962.5
ERROR SUM OF SQUARES = 175219825

REGRESSION MEAN SQUARE = 347474981.3
ERROR MEAN SQUARE = 7300826.045

F-RATIO = 47.59392692

COEFFICIENT OF DETERMINATION (R-SQUARE) = 0.798637200

STANDARD ERROR OF THE ESTIMATE = 2702.004079

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